

Chapter 10 — Equivalent Ratios and Ratio Tables — Teacher Guide

Chapter at a glance

Lesson duration	4 lessons, 45–60 minutes each (see pacing table below)
Materials	Two-color counters, grouped strips, recipe cards, scale-factor cards, ratio-table templates, double number lines, context cards, the three chapter diagrams
Launch question	“A drink mix uses water:concentrate = 3:5. Do 6:10, 9:15, and 6:8 all describe the same mixture?”
Key representation	The ratio table with a scale-factor column — keep one visible throughout instruction
Anticipated responses	Correct: identifies one common factor and applies it to both parts. Partial: scales correctly but can’t name the factor, or scales only one part. Incorrect: adds the same number to both parts (for example, “3:5 becomes 6:8”).
If the readiness check fails	Reteach ratio order and labels from Chapter 9 before starting the Launch; do not skip the equal-groups counters step
Talk prompt	“What factor changed the first part? Does the second part use that same factor?”
Exit ticket	5 items — complete a missing term, test equivalence, simplify, explain why addition fails, state a table check (see Section 13)
Reteach / extend	Reteach: whole-number scale factors 2, 3, 4 only, with equal-group cubes. Extend: fractional scale factors, multi-row tables, proving equivalence multiple ways.

Purpose and pacing

Chapter 10 develops the multiplicative structure that underlies proportional reasoning. Ratio tables should be used as reasoning tools, not answer lists. Each row should have a visible scale factor or a clear connection to a previous row.

Lesson	Focus	Suggested time
Lesson 1	Readiness, launch, equal groups, and scaling	45–60 minutes
Lesson 2	Ratio tables, double number lines, and missing terms	45–60 minutes

Lesson	Focus	Suggested time
Lesson 3	Context scaling, simplification, misconceptions, and Worksheet A	45–55 minutes
Lesson 4	Mixed practice, Tripwire, Exit Ticket, and targeted reteaching	50–65 minutes

Materials

Prepare two-color counters, grouped strips, recipe cards, scale-factor cards, ratio-table templates, double number lines, and context cards. Use physical grouping before introducing abstract completion of $a:b = c:d$.

Teacher launch guidance

Begin with 3:5 and ask students to create a double batch, triple batch, and half batch. Have students explain what remains unchanged. Then display 3:5 and 6:8 and ask students to defend or reject the claim with a model.

When students add instead of multiply, do not begin with a rule. Ask them to build the groups. If the first quantity doubles, the second quantity must also double. The visual contradiction makes the multiplicative requirement meaningful.

Teacher prompts

Purpose	Prompt
Identify scale	“What factor changed the first part?”
Preserve relationship	“What must happen to the second part?”
Use context	“What does one group represent?”
Complete a table	“Which known row can you scale from?”
Simplify	“What common factor divides both parts?”
Diagnose additive error	“Did the same multiplier act on both quantities?”
Check equivalence	“Can both ratios simplify to the same form?”

Differentiation

Students needing concrete support

Use equal groups of counters and physical recipe cards. Keep the original ratio visible, and create one full group at a time. Use scale factors 2, 3, 4, and 5 before fractional or reverse scaling.

Students needing language support

Use the frames:

- “The original ratio compares _____ with _____.”
- “The scale factor is _____ because _____.”

- “I multiply both parts by _____.”
- “The ratios are equivalent because _____.”
- “The relationship changes when _____.”

Students ready to extend

Ask students to scale by fractions, compare multiple equivalent tables, analyze maps and mixtures, and prove equivalence by simplification, scale factor, and double-number-line reasoning.

Formative assessment

Record whether students identify a common factor, apply it to both parts, preserve order, use context units, and explain why the relationship stays the same. A correct missing term obtained without a scale explanation should be treated as partial evidence on a reasoning task.

Revision Support: Expected Ideas and Teacher Moves

Use this compact guide with the new Cumulative Connection and Strategy Studio tasks. Accept equivalent representations when the relationship, units, and reasoning are correct.

Cumulative Connection: Complete $3:4 = 15:__$. State the scale factor from $6:8$ to $24:32$. Decide whether $4:7$ and $12:21$ are equivalent. Explain why a ratio table can include fractions or decimals.

Strategy Studio guidance: Expected ideas: $3:4 = 15:20$; $6:8$ to $24:32$ uses factor 4; $8:12$ and $14:21$ both simplify to $2:3$; the broken table fails at 6 to 16 because the factor is not 2.5. Accept fractional scale factors as valid proportional reasoning.

Transfer Bridge Teacher Guidance

Use the four tasks to distinguish imitation from transfer. Ask students to name the relationship, keep units visible, compare representations, and explain or repair an error. Accept equivalent methods when the structure and interpretation are correct.

Expected guidance: Familiar: $3:4$ scaled by 5 gives $15:20$. Altered: both $8:12$ and $14:21$ simplify to $2:3$. Non-routine: the table breaks at 6 to 16 because $6 \times 2.5 = 15$, not 16. Repair: one common factor must apply to both quantities.

Record whether the difficulty is classification, representation, operation, computation, units, interpretation, or checking. Do not assign more routine practice until the source of the error is identified.

Discussion and evidence note

Ask students to name the relationship before calculating, compare at least two representations when appropriate, and revise an explanation after hearing another strategy. Record whether the difficulty is classification, representation, operation, computation, units, or checking.

Answer Key

1. It compares the first named quantity with the second.
2. $4:6$.

Ready-to-Learn Check

3. 2:3.
4. Yes.
5. Multiplication by a common factor preserves the relationship; addition generally does not.
6. 6 cups of water.
7. The factor applied to both parts.

Worksheet A

1. **4:6.** Likely wrong answer: 4:5 (multiplies only the first part by 2, leaving the second part unchanged or altered incorrectly). Misconception: scaling only one quantity instead of both. Next move: "what must happen to the second part when the first part doubles?"
2. **12:15.** Likely wrong answer: 7:8 (adds 3 to both parts instead of multiplying both by 3). Misconception: treating equivalence as an additive shift rather than a multiplicative scale. Next move: "does adding the same number to both parts keep the relationship the same?"
3. **6.** Likely wrong answer: 7 (adds the difference between 2 and 4, which is 2, to the second part instead of multiplying by the scale factor). Misconception: additive rather than multiplicative reasoning. Next move: "what factor changed 2 into 4? Apply that same factor to 3."
4. **4.** Likely wrong answer: 7 (adds the difference between 5 and 10, which is 5, to the second part instead of multiplying by the scale factor). Misconception: same additive error as #3. Next move: "what factor changed 5 into 10? Apply that same factor to 2."
5. **Yes, factor 2.** Common flaw: says "yes" without identifying or checking the actual scale factor. Next move: ask for the specific number that multiplies both parts.

6. **No; the factors do not match.** Likely wrong answer: "Yes," assuming any two ratios with similar-looking numbers must be equivalent. Misconception: not checking that one consistent factor applies to both parts. Next move: "what factor takes 3 to 6? Does that same factor take 4 to 7?"
7. **2:3.** Likely wrong answer: 4:6 (divides by 2, a common factor, but not the greatest common factor of 8 and 12). Misconception: stopping before reaching the simplest form. Next move: "is there a larger number that still divides both 8 and 12 evenly?"
8. **The first part uses a factor of 2 (2 → 4), but the second part uses a factor of 1.4 (5 → 7); the factors don't match, so the ratios are not equivalent.** Common flaw: notices the ratios "look different" without identifying the actual mismatched factors. Next move: ask for the specific factor each part would need.
9. **Two groups of 3 and 5.** Common flaw: draws groups of unequal or mismatched sizes that don't clearly represent two identical copies of 3:5. Next move: check that both groups are visibly identical.
10. **Factor 4.** Likely wrong answer: describes the change as "add 9" ($12 - 3 = 9$) instead of identifying a single multiplicative factor. Misconception: additive rather than multiplicative reasoning. Next move: "what number multiplies 3 to get 12? Does that same number work for 5 and 20?"

Worksheet B

1. **Examples: 6:8, 9:12, 12:16.** Likely wrong answer: 7:8 (adds a fixed amount to both parts instead of multiplying by a scale factor).

- Misconception: additive rather than multiplicative reasoning. Next move: “what number multiplies 3 to give your first part? Does that same number multiply 4 correctly?”*
2. **2:3.** Likely wrong answer: 5:10 (divides by a common factor but not the greatest one, or divides only one part). *Misconception: incomplete simplification. Next move: “what is the greatest number that divides both 10 and 15?”*
 3. **20.** Likely wrong answer: 17 (adds the difference between 3 and 12, which is 9, to the second part instead of multiplying by the scale factor). *Misconception: additive rather than multiplicative reasoning. Next move: “what factor changed 3 into 12? Apply it to 5.”*
 4. **28.** Likely wrong answer: 4 (multiplies 4 by 1 instead of finding the factor that turns 1 into 7). *Misconception: not identifying which part changed or by what factor. Next move: “which number is known on both sides — the 4:1, or the 7?”*
 5. **Yes, factor 2.** Common flaw: says “yes” without identifying or checking the actual scale factor. *Next move: ask for the specific number that multiplies both parts.*
 6. **No.** Likely wrong answer: “Yes,” assuming any two ratios with similar-looking numbers must be equivalent. *Misconception: not checking that one consistent factor applies to both parts. Next move: “what factor takes 7 to 14? Does that same factor take 9 to 20?”*
 7. **18 cups.** Likely wrong answer: 9 cups (adds the difference between 4 and 12, which is 8, to the flour amount instead of multiplying by the scale factor). *Misconception: additive rather than multiplicative reasoning. Next move: “what factor changed 4 oats into 12 oats? Apply it to the 6 cups of flour.”*
 8. **The first part changed from 3 to 6, which is multiplication by 2. To preserve the ratio, the second part must also be multiplied by 2: $8 \times 2 = 16$. The correct equivalent ratio is 6:16.** Likely wrong answer: agrees that 6:11 is correct, since both parts increased by the same amount (3). *Misconception: additive rather than multiplicative reasoning — the chapter’s central error. Next move: “does adding 3 to both parts use the same multiplicative factor on each?”*
 9. **2:7, 4:14, 6:21, 10:35.** Likely wrong answer: uses a different factor for each row instead of matching the stated scale factors 1, 2, 3, and 5. *Misconception: not applying the specified factor consistently to both parts. Next move: “for scale factor 3, what do you multiply 2 and 7 by?”*
 10. **Factor 4.** Likely wrong answer: describes the change additively ($32 - 8 = 24$) instead of identifying a single multiplicative factor. *Misconception: additive rather than multiplicative reasoning. Next move: “what number multiplies 5 to get 20? Does that same number work for 8?”*
 11. **Divide both by 12 to get 2:3.** Likely wrong answer: divides only one part, or uses a smaller common factor and stops before reaching 2:3. *Misconception: incomplete simplification. Next move: “what is the greatest number that divides both 24 and 36?”*
 12. **Answers vary but must preserve 4 for every 7.** Common flaw: gives a context but the actual numbers used

don't maintain a 4:7 relationship. Next move: check the context's numbers actually simplify or scale to 4:7.

Worksheet C

1. **2:5, 4:10, 6:15, 10:25.** Likely wrong answer: uses a different factor for the yogurt column than the bananas column, breaking the relationship (for example, 2:5, 4:9, 6:13). Misconception: not applying one consistent factor to both parts. Next move: "what factor takes 2 bananas to 4? Does the yogurt column use that same factor?"
2. **35 L.** Likely wrong answer: 19 L (adds the difference between 3 and 15, which is 12, to the yellow amount instead of multiplying by the scale factor). Misconception: additive rather than multiplicative reasoning. Next move: "what factor changed 3 litres of red into 15 litres? Apply it to the 7 litres of yellow."
3. **12 adults.** Likely wrong answer: 8 (confuses which number is the scale factor, or divides in the wrong direction). Misconception: not identifying which known quantity corresponds to which ratio part. Next move: "96 students corresponds to which number in 1:8? What factor gets you there?"
4. **10:16, 15:24, and 25:40.** Likely wrong answer: includes 20:33, which scales the first term correctly ($5 \times 4 = 20$) but not the second ($8 \times 4 = 32$, not 33). Misconception: checking only one term's scale factor and assuming the other matches. Next move: "does one common factor turn 5:8 into 20:33?"
5. **The first part changes from 2 to 8 by factor 4, so the second should be 28, not 29.** Common flaw: identifies that 29 is wrong but doesn't state the correct value or the factor that produces it. Next move: require both the correct value and the factor used to find it.
6. **20 cups flour.** Likely wrong answer: 11 cups (adds the difference between 2 and 8, which is 6, to the flour amount instead of multiplying by the scale factor). Misconception: additive rather than multiplicative reasoning. Next move: "what factor changed 2 cups sugar into 8 cups? Apply it to the 5 cups of flour."
7. **Examples: 2:8, 3:12, 4:16.** Likely wrong answer: uses a different factor for each part within the same ratio (for example, 2:8 correctly, but then 3:11 instead of 3:12). Misconception: not applying one consistent factor to both parts of a single ratio. Next move: "for a scale of 1:4, if the first part is 3, what must the second part be?"
8. **Different factors distort the relationship.** Common flaw: says the result is "wrong" without explaining specifically that using two different factors breaks the single-factor requirement. Next move: ask what factor was applied to each part separately.
9. **Answers vary.** Common flaw: the four rows don't all share one consistent scale-factor pattern. Next move: check that every row can be reached from the first by a single multiplicative factor.
10. **Simplify both ratios and compare their simplest forms.** Common flaw: says to "simplify" without explaining that matching simplest forms is what confirms equivalence. Next move: ask what it means if the simplified forms don't match.

11. **20 cups of butter. The number of ovens (2) and the baking time (45 minutes) were not needed to find the amount of butter.** *Likely wrong answer: works the 2 ovens or 45 minutes into the calculation somehow. Misconception: assuming every number in a word problem must be used. Next move: "what does the question actually ask for? Which numbers answer that?"*
12. **Answers vary; the scale factor is 5 ($4 \times 5 = 20$ and $6 \times 5 = 30$), meaning the situation was scaled up to five times its original size.** *Common flaw: gives a context but doesn't state the scale factor itself, or states it as an amount added rather than a multiplier. Next move: ask "what single number multiplies both 4 and 6 to reach 20 and 30?"*
3. Divide 18 by 6, so divide 12 by 6: 2:3.
4. Example: 2:3 becomes 6:7 by adding 4, but 2:3 scaled by 3 is 6:9.
5. Multiplying both parts by the same fraction preserves the ratio, though the context may require fractional quantities.
6. Answers vary; the scale relationship remains 3 units of the first for every 4 of the second.

Tripwire and Exit Ticket

Tripwire: 8:14.

Exit Ticket: 49; yes, both simplify to 2:3; 3:4;

addition does not apply one common multiplicative factor;

checks include using the same scale factor in both columns or simplifying both ratios to the same form.

Extend Your Thinking

1. Divide both by 10 to get 2:5.
2. Factor 5, so 35:55.

